Physics II ISI B.Math Final Exam : April 29, 2010

Total Marks: 100 Answer any five questions

1. a) A litre of nitrogen gas at atmospheric pressure and 0° C in a rigid cylinder is raised to 100° C by placing it in contact with an infinite reservoir at 100° C. What are the changes in the entropy of the nitrogen gas and the universe? The specific heat at constant volume of nitrogen is $\frac{5}{2}R$ per mole and the molar volume is 22.4 litres. R = 8.31 J/mol-K (14)

b) Assuming that one wall of this cylinder is allowed to act like a piston, describe a means of raising the gas temperature to 100° C (with the final volume of 1 litre) such that the entropy change $\Delta S = 0$ for the universe. (6)

2. Two identical objects A and B are mechanically and thermally isolated from the rest of the world. Their initial temperatures are $\tau_A > \tau_B$. Each object has heat capacity C (the same for both objects) which is independent of temperature.

(a) Suppose the objects are placed in contact with each other and allowed to come to equilibrium. What is their final temperature ? How much entropy is created in the process ? How much work on the outside world is done in the process? (8)

(b) Instead suppose objects A (temperature τ_A) and B (temperature $\tau_B < \tau_A$) are used as the high and low temperature reservoirs of a heat engine. The engine extracts energy from object A (lowering its temperature), does work on the outside world and dumps waste heat to object B, (raising its temperature). When A and B are at the same temperature the process is finished. Suppose this heat engine is the most efficient heat engine possible. What is the final temperature of the objects ? How much entropy is created in this process ? How much work is done on the external world in this process? (12)

3. Consider the earth's atmosphere as an ideal gas of molecular weight μ in a uniform gravitational field. Let g denote the acceleration due to gravity.

(a) If z denotes the height above sea level, show that the change in atmospheric pressure with height is given by

$$\frac{dp}{p} = -\frac{\mu g}{RT}dz$$

where T is the absolute temperature at height z. (4)

(b) If the decrease in pressure in (a) is due to adiabatic expansion, show that

$$\frac{dp}{p} = \frac{\gamma}{\gamma - 1} \frac{dT}{T}(4)$$

(c) From (a) and (b) calculate $\frac{dT}{dx}$ in degrees per kilometre. Assume the atmosphere to consist of mostly nitrogen, for which $\gamma = 1.4$. (4)

(d) In an isothermal atmosphere at temperature T, express the pressure p at height z in terms of the pressure at sea level. (4)

(e) In the sea level pressure and temperature are p_0 and T_0 respectively, and the atmosphere is regarded as adiabatic as in part (b), find again the pressure p at height z.(4)

4. (a) In a double slit experiment, the wavelength λ of a light source is 405 nm, the slit separation d is 19.44 μ m and the slit width a is 4.050 μ m. Consider both the interference and diffraction through the slits. How many bright interference fringes are within the central peak of the diffraction maxima? How many bright fringes are within either of the first side peaks of the diffraction envelope? If we increase the wavelength of light to 550 nm, do the width of the central diffraction peak and the number of interference fringes within the peak increase, decrease or remain the same? (14)

(b) A diffraction grating has 1.26×10^4 rulings uniformly spaced over a width of 25.4 mm. It is illuminated at normal incidence by light of wavelength 589.00 nm. At what angle does the first order maximum occur (on either side of the centre of the diffraction pattern?(6)

5. (a) A glass plate of index 1.50 is to be a non-reflecting surface . What should be the thickness of the transparent surface coating material of refractive index 1.25 for green light of wavelength 5500 Å to achieve this ?(5)

(b) Consider the Newton's Ring arrangement, where a plano convex lens of very large radius of curvature R rests on a flat glass plate. Monochromatic light of wavelength λ is incident normally on the arrangement and alternate dark and bright rings are observed in the interference pattern. Show that the radius r of the mth bright ring is proportional to $\sqrt{m + \frac{1}{2}}$. Is the central spot dark or bright ? Explain why. If the central spot turns out to be dark (bright), can you suggest a modification in the arrangement so that the central spot changes to bright(dark)?(8)

(c) An oil drop (n = 1.20) floats on water (n=1.33) surface and is observed from above by reflected light. (i) Will the outer(thinnest) region of the drop correspond to a bright or dark region ?(ii) Approximately how thick is the film where one observes the third blue region from outside the drop ? (iii) Why do the colours gradually disappear as the thickness of the drop becomes larger? (7)

6. A double slit interference pattern is produced by light of wavelength λ passing through two slits of unequal widths, say $w_1 = 20\lambda$ and $w_2 = 40\lambda$, their centres being 1000λ apart. If the observation is made on a screen at a distance L very far away from the slits , i.e., $L \gg 1000\lambda$, determine the following features:

(a) separation δx between adjacent maxima (4)

(b) widths Δx_1 and Δx_2 of the diffraction patterns of the two slits individually (i.e., distance between first zeroes) (4)

(c)Hence the number of fringes produced by the overlap of these two central maxima. (4)

(d)The intensity ratio between the intensity maximum and minimum in the center of the pattern. (4)

(e)An analytic expression for the intensity on the screen as a function of x when x = 0 is at the exact centre of the pattern.(4)